The Shape and Structure of Musical Contour Space

David Kant and Larry Polansky, University of California Santa Cruz

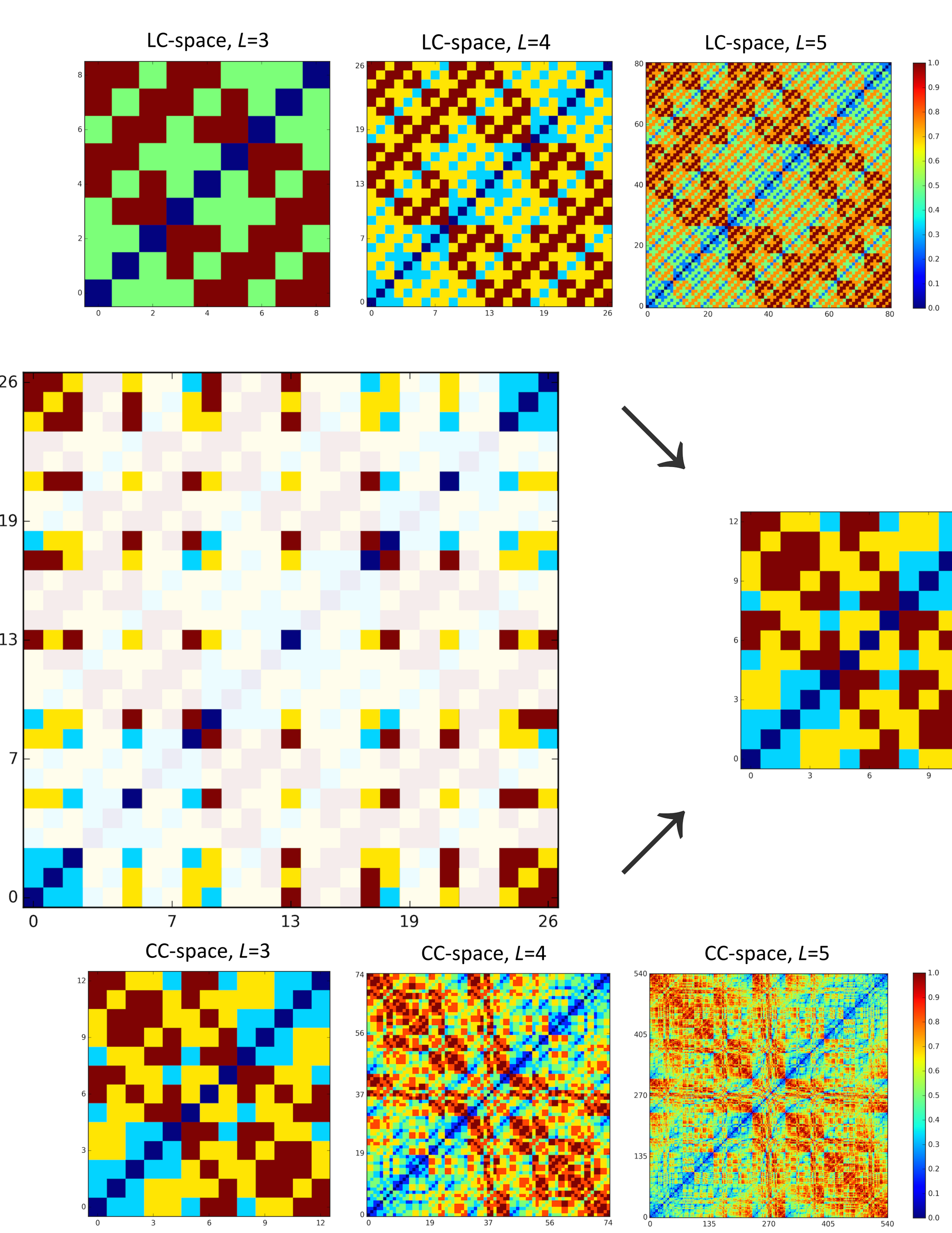
Overview: Using the techniques of *n-ary contour, full rank, and basis coordinate space* we express all possible CCs for any *n (resolution)* and any *L (length)*, and by extension, *all possible morphologies of any length with elements of arbitrary magnitudes*, formally collapsing the distinction between *contour* and *morphology into* a unified representation of “contour/morphological space” on the unit hypersphere.

Definitions

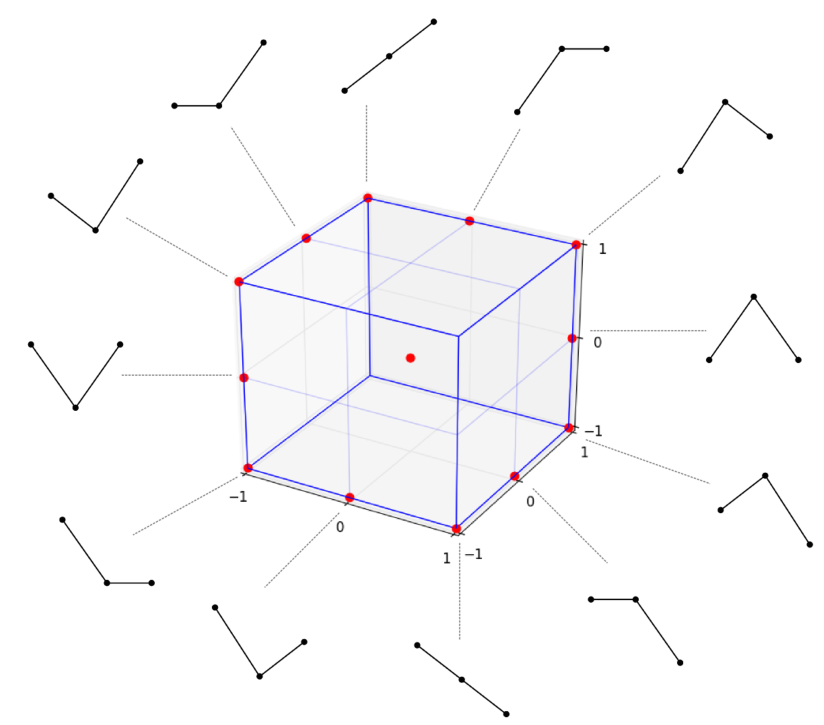
- a *contour* is a vector of directional relationships “up/down/ equal”
- *linear contour* (LC) characterizes relationships between adjacent elements
- *combinatorial contour* (CC) is the network of relationships between all pairs

CC-Space as a Lumpy Subset of LC-Space

- Any CC is also an LC (representing *morphs* of different lengths) — both are written as uni-dimensional ternary-valued vectors
- Not all LCs are CCs. CCs are restricted by 1) allowable lengths of 2nd-order binomial coefficients and 2) *transitivity* of directional relationships
- LCs that are not CCs are “holes” in CC-space
- LC-space is *a closed linear space* — distance is translational under OCD (Hamming metric)
- Due to holes, CC-space is *not* a closed linear space — distance in CC-space is irregular and not translational as in LC-space
- CC-space structure is inherited from LC-space, but not all features of LC-space hold for CC-space



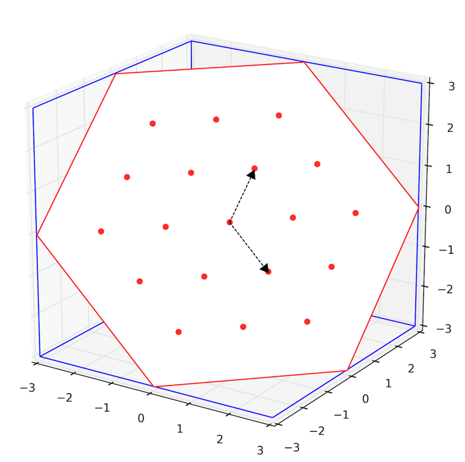
dissimilarity matrices show the metric structure of LC- and CC-space



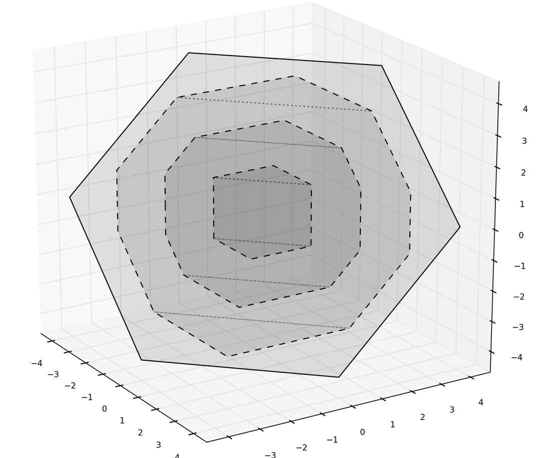
geometric representation of the 13 possible CCs of L=3

Full Rank and n-ary Contour Resolution

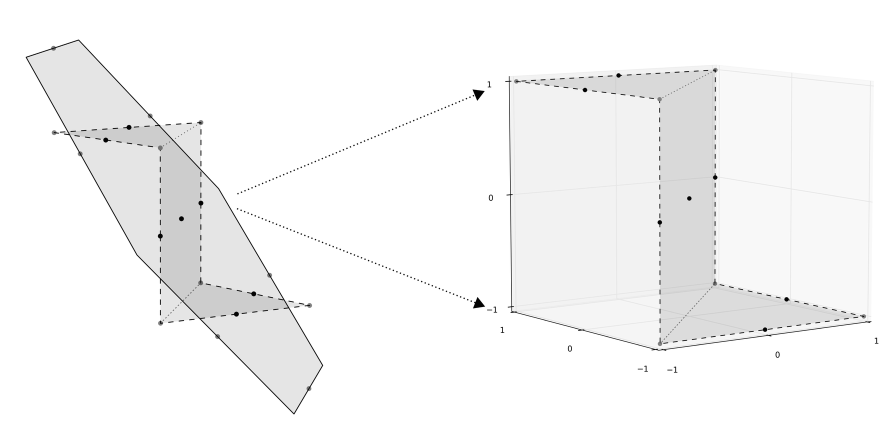
- Some *possible* CCs are *full rank* and *do not compress ranking values*; other CCs are not full rank and *do compress* ranking values
- In full rank CCs all magnitudes of pairwise inter-element differences (or *deltas*) between *morph normal form* ranked elements are described *without compression* (additive property holds)
- The *resolution n* of contour representation (e.g. ternary, n = 1) may not fully express the normal form ranking of a morph’s elements
- Any possible CC not full rank for some n, is full rank for some greater n
- For a full rank CC of length L, n must be sufficient to express the maximum rank (L-1)



full rank CC plane in L=3



n-ary folding of the CC plane



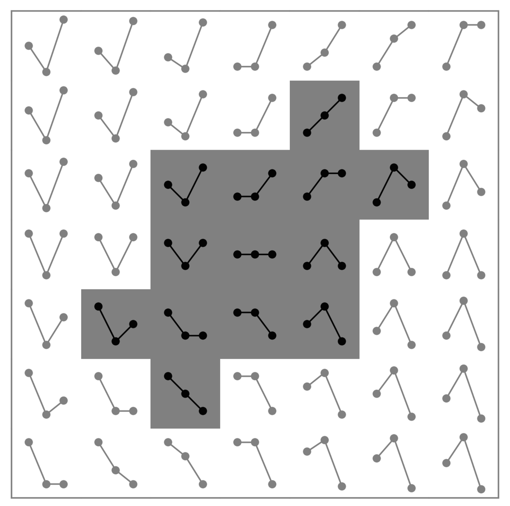
folding the CC plane to the ternary cube

Extra credit: What is the # of CCs for given L, n (n odd)?

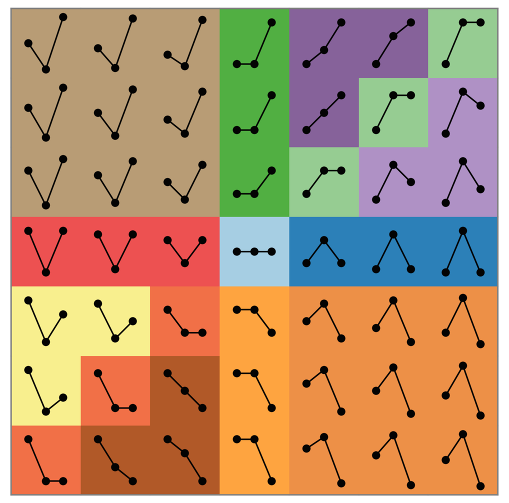
Basis-Space

- Full rank CCs of L = 3 *form a plane* in the 3-dimensional cube, a *linear subspace*. This linearity allows us to clarify the *structure of CC-space*
- We can consider CCs of a given length as a *closed vector space*, and determine *bases* for this space, greatly reducing dimensionality, and producing a more compact visualization and representation of *distance* (structure)
- *Basis-space* is a representation of CC-space which comprises *all and only full rank CCs*

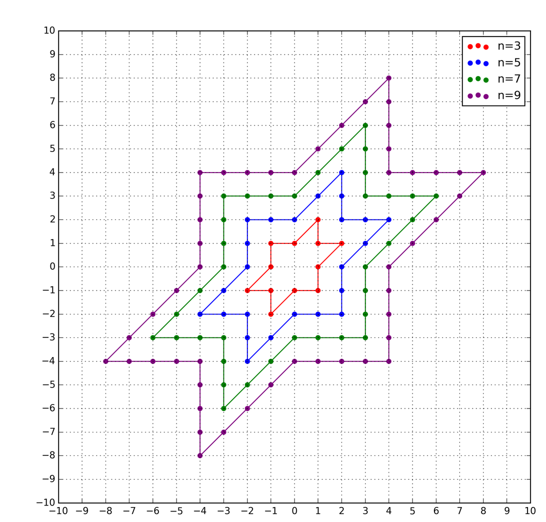
2-d basis-space *L=3*



13 ternary contours (n=3) normal form contours in grey



equivalence regions of the 13 ternary contours as n increases

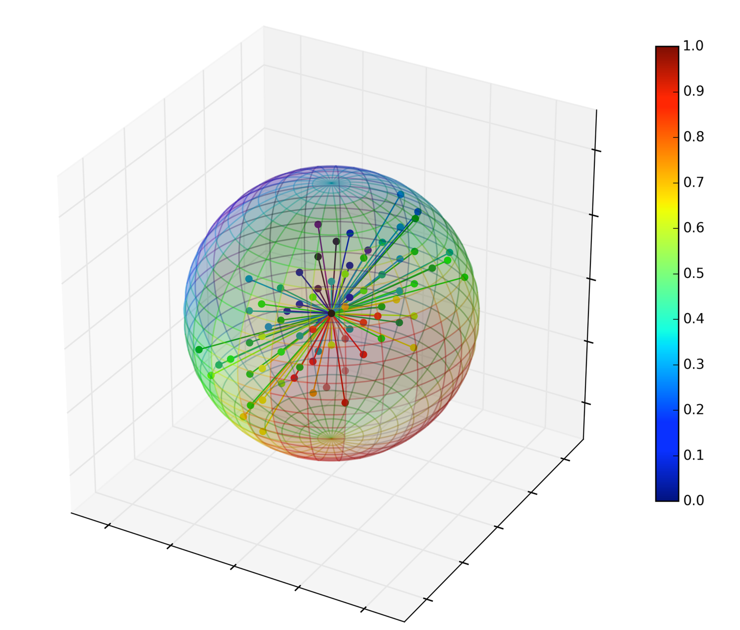


basis-space stellated form for n-ary contours for n = 3,5,7,9 (L = 3)

- Through a *change of coordinate systems*, full rank CCs are represented by a set of *basis vectors* which combine arithmetically to form *all possible full rank CCs*, implicitly distinguishing possible from impossible CCs, and describing all possible CCs and their proximities in minimal dimensionality
- Basis-space provides a *set of basis vectors B* which expresses full rank CCs as weighted sums of basis vectors $b_1, b_2, \ldots, b_{L-1}$ together with scalars $a_1, a_2, \ldots, a_{L-1}$. These basis vectors become the axes of a new coordinate system: *basis-space*.

The Unit Sphere

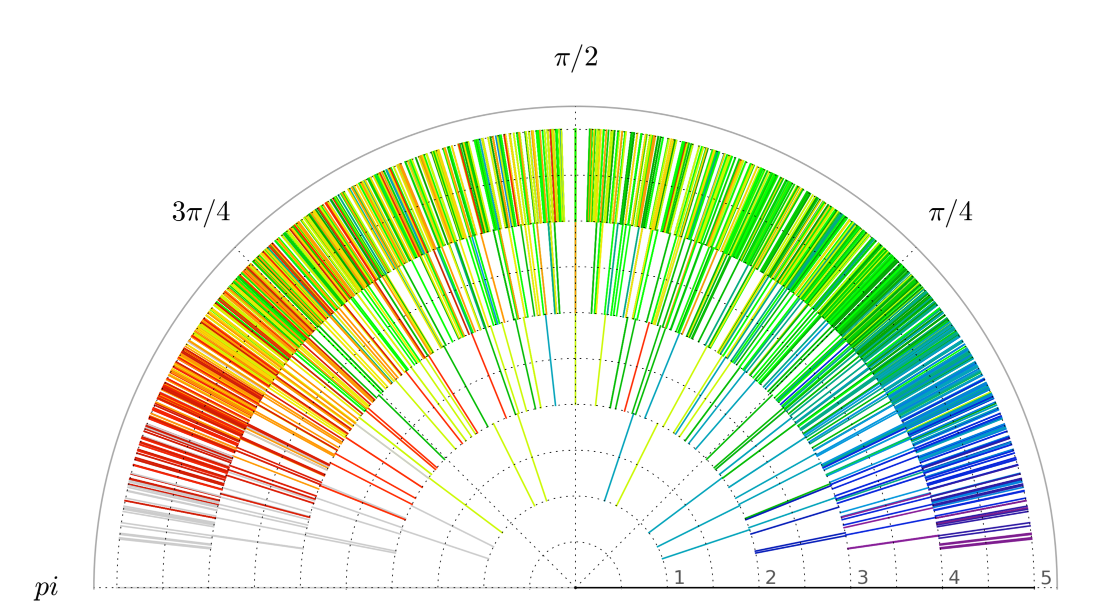
- *Normalizing* basis coordinates to their maxima inverts the hyperplane and yields a generalized *continuum* of contour and morphology, forming a b-dimensional *unit hypersphere,* a metric space of CCs of all n and L
- All possible CCs for any n and any L can be expressed — by extension, *all possible morphologies of any length with elements of arbitrary magnitudes*
- This formally collapses the distinction between *contour* and *morphology*



3-d basis-space (L=4) inverted to unit sphere by normalizing basis coordinates

A Unified Metric Space of Contour and Magnitude

- Contours of varying lengths are represented in the same dimensionality by converting basis-coordinates into polar coordinates of high dimensional angle (*cosine distance*) and rank, creating a *new formulation of distance* in CC- (thus morph) space.
- Distance between CCs can be taken along dimensions of angle and/or rank, forming a metric space inclusive of CCs of all n and L (thus, all morphs).



2-d projection of the 4-d unit hypersphere (L=3,4,5) in which angle is cosine distance between basis coordinates and magnitude is rank n